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Preface

The North American Electric Reliability Corporation (NERC) is a not-for-profit international regulatory authority whose mission is to assure the reliability and security of the bulk power system (BPS) in North America. NERC develops and enforces Reliability Standards; annually assesses seasonal and long-term reliability; monitors the BPS through system awareness; and educates, trains, and certifies industry personnel. NERC’s area of responsibility spans the continental United States, Canada, and the northern portion of Baja California, Mexico. NERC is the Electric Reliability Organization (ERO) for North America, subject to oversight by the Federal Energy Regulatory Commission (FERC) and governmental authorities in Canada. NERC’s jurisdiction includes users, owners, and operators of the BPS, which serves more than 334 million people.

The North American BPS is divided into eight Regional Entity (RE) boundaries as shown in the map and corresponding table below.

The North American BPS is divided into eight RE boundaries. The highlighted areas denote overlap as some load-serving entities participate in one Region while associated transmission owners/operators participate in another.

<table>
<thead>
<tr>
<th>Acronym</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>FRCC</td>
<td>Florida Reliability Coordinating Council</td>
</tr>
<tr>
<td>MRO</td>
<td>Midwest Reliability Organization</td>
</tr>
<tr>
<td>NPCC</td>
<td>Northeast Power Coordinating Council</td>
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<tr>
<td>RF</td>
<td>Reliability First</td>
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<tr>
<td>SERC</td>
<td>SERC Reliability Corporation</td>
</tr>
<tr>
<td>SPP RE</td>
<td>Southwest Power Pool Regional Entity</td>
</tr>
<tr>
<td>Texas RE</td>
<td>Texas Reliability Entity</td>
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<tr>
<td>WECC</td>
<td>Western Electricity Coordinating Council</td>
</tr>
</tbody>
</table>
Executive Summary

NERC assesses current and future adequacy and operational reliability of the North American Bulk Power System (BPS) through seasonal, long-term and short-term special assessments. These assessments inform policy makers and regulators of the emerging issues and potential concerns by identifying notable trends impacting the BPS in the United States, Canada, and the portion of Baja California Norte, Mexico.

The electric power industry is undergoing significant and rapid change. As a result, several emerging key issues have been identified as having the potential to increase risks to reliability. These issues include resource adequacy deficiencies, changes in the resource mix leading to a higher percentage of dependency on one fuel, nuclear and coal power plant availability uncertainty, inability to provide Essential Reliability Services, management of Distributed Energy Resources, predicting the performance of Variable Energy Resources, fuel supply security, and transmission aging and extended transmission outages. Over the years, NERC has devoted a great deal of efforts in assessing the reliability of North American BPS for example the Long Term Reliability Assessment (LTRA) and Seasonal Assessments. These assessments have largely relied on a traditional deterministic reserve margin measurement to assess risks to reliability. NERC recognizes that these emerging issues are highly variable and uncertain and they can have an effect on the traditional resource adequacy assessments. NERC is considering the value of implementing more probabilistic approaches to measuring the BPS resource and transmission adequacy and evaluating whether probabilistic approaches should be used permanently in resource adequacy/reliability assessments.

The NERC Probabilistic Assessment Working Group (PAWG) was assigned the task to review the use of probabilistic studies in assessing these emerging reliability risks and producing this report. This report covers the work done by NERC, the Planning Committee (PC), the Reliability Assessment Subcommittee (RAS), and the PAWG in supporting this needed review.

Objectives

The objectives of this report are:

- Develop a collective understanding of existing applications of probabilistic techniques used for reliability assessments and planning studies.
- Identify emerging reliability issues for which probabilistic studies are likely to provide significant insights
- Review existing Reliability Risk Metrics (RRMs) and provide a common understanding of their definitions and use and recommendations for future enhancements and applications.
- Identify commonalities to inform industry on the applications of probabilistic reliability metrics.
- Provide guidance on the development of probabilistic methods for ensuring resource adequacy and reliability to allow better risk-informed decisions for planners and policy makers in the face of increasing uncertainty of supply and demands, on the bulk power system.

The foundation of the report is based on results from a NERC survey on probabilistic studies, as well as data and information gathered by NERC from Regions and Assessment Areas.

Survey Objectives:

- Review the ongoing probabilistic analyses and future plans for further insights into resource adequacy assessment.
- Understand the choice of probabilistic methods, tools, and selection of acceptable reliability levels used by NERC regions and the industry at large to assess resource and transmission adequacy.
- Show the need to expand probabilistic studies to help assess emerging reliability issues that have an impact on BPS reliability.
Executive Summary

- Explore probabilistic approaches used that provide further insights into how to best establish adequate reserve margins amidst a BPS undergoing unprecedented changes.
- Identify how members of industry define, and apply Reliability Risk Metrics (RRMs).
- Explore applications of commonly used RRMs and how each RRM can measure different aspects of a system’s reliability, such as frequency, duration and magnitude of loss of load, depending on how the metric is defined and applied.
- Provide recommendations on the application of commonly used RRMs in assessing system adequacy.

Key Findings

- There are variations in how a reliability criterion is defined and interpreted in existing practices in the assessment areas across the U.S. and Canada.
- The majority of entities in North America conducting resource adequacy studies primarily use Loss of Load Expectation (LOLE) metric to establish a single resource adequacy on criterion. In turn, the LOLE RRM generally helps inform integrated resource planning, market-based resource procurement, generator interconnection queue projects, and other planning activities.
- About one third of survey respondents use the Expected Unserved Energy (EUE) metric for assessing reliability. EUE provides insight to the impact of energy limited resources on a system’s reliability, particularly in systems with growing penetration of such resources. Examples of such energy limited resources include:
  - Demand Response programs, which can be modeled as resources with specific contract limits including hours per year, days per week, and hours per day constraints,
  - Energy Efficiency programs, which can be modeled as reductions to load, with an hourly load shape impact
  - Distributed Energy Resources (DERs), such as behind the meter solar PV, which can be modeled as reductions to load, with an hourly load shape impact
- The choice of probabilistic methods and selection of acceptable adequacy levels are still matters of judgment and differ from Region to Region and from assessment area to assessment area and even utility to utility in some cases.
- Most Assessment Areas are already using or are considering probabilistic approaches to assess emerging reliability issues.
- There is a recognized need to support probability-based resource adequacy assessment resulting from the changing resource mix with significant increases in variable and energy-limited resources (intermittent in nature), changes in net demand profiles resulting in the shifting of the hour of the peak demand, and other factors can have an effect on resource adequacy.
- A number of issues based on industry survey results are out of the PAWG scope of work and therefore, are not discussed in this report. These issues are: 1) operational concerns such as unit commitment, over-generation and dispatch issues, 2) Essential Reliability Services issues such as VER’s capacity credit evaluation, ramping, flexibility, and regulations and 3) potential resource upgrades.

General Recommendations

The Reliability Assessment Subcommittee (RAS) agrees with the PAWG recommending the following:

- Entities may leverage other metrics and factors in their criteria development to determine a sufficient reserve margin to maintain an adequate level of system reliability, especially for systems with a diverse generation mix and variable energy resources.
Executive Summary

• NERC should continue to incorporate more probabilistic approaches into its assessments, and continue to review and provide guidance on the development of probabilistic methods for ensuring resource adequacy and reliability.
• NERC should continue conducting periodic reviews on RRMs and criteria used to assure they are clear and properly structured for existing and emerging risks.
• As entities and system planners identify emerging reliability issues or large changes on their system (e.g. change in size, resource mix, etc.), they should evaluate whether the incorporation of additional RRMs could improve their assessment of risks to reliability.

Detailed Recommendations on Reliability Risk Metrics (RRMs)
The Reliability Assessment Subcommittee (RAS) agrees with the PAWG recommending the following:

Loss of Load Hours (LOLH)
PAWG recommends the use of LOLH RRM using all hours, rather than just peak periods for both small and large systems. It can be evaluated over seasonal, monthly or weekly study horizons. LOLH does not inform of the magnitude or the frequency of loss of load events, but it is used as a measure of their combined duration. LOLH is applicable to both small and large systems and is relevant for assessments covering all hours (compared to only the peak demand hour of each season). LOLH provides insight to the impact of energy limited resources on a system’s reliability, particularly in systems with growing penetration of such resources. Examples of such energy limited resources include:
  • Demand Response programs, which can be modeled as resources with specific contract limits including hours per year, days per week, and hours per day constraints,
  • Energy Efficiency programs, which can be modeled as reductions to load, with an hourly load shape impact
  • Distributed resources, such as behind the meter PV, which can be modeled as reductions to load, with an hourly load shape impact

Loss of Load Expected Events (LOLEV)
PAWG recommends LOLEV to be used alongside other metrics specified in this report when evaluating capacity planning decisions, more for systems where planners are concerned about the potential for multiple loss of load events in a single day.

Loss of Load Expectation (LOLE)
For LOLE RRM, PAWG recommends:
  • Entities evaluate all hours of a given time period when calculating LOLE, especially considering the impact a changing resource mix (particulary DERs and VERs) is having on the daily load distributions of many areas across the BPS.
  • Entities to report the time period and hours associated with their LOLE calculation and the reasoning behind their approach as for instance, the LOLE evaluated on just the daily peak hours will always be equal to or less than an LOLE based on all hours.

Expected Unserved Energy (EUE)
With the changing generation mix and to make EUE a more effective metric, PAWG recommends that:
  • Hourly EUE values should be reported for every month or year (i.e., 24 data points) as this is the only metric which considers magnitude of loss of load events.
  • System planners estimate the cost and impact of the loss of load events using EUE as it is a useful measure in estimating the size of loss of load events and can be used as basis for reference reserve margin to determine capacity credits for variable energy resources.
  • For extreme weather conditions and common mode failure events, PAWG recommends using EUE RRM as this measure quantifies events impacts on system reliability.
Introduction

NERC recognizes that such factors as the changing resource mix, shifting demands and others can have a significant effect on resource adequacy. As a result, NERC is incorporating more probabilistic approaches and other ongoing analyses to provide further insights on how to best establish adequate reserve margins and analyze other reliability issues. While NERC has historically gauged resource adequacy using deterministic planning reserve margins, it is now exploring the expanded use of probabilistic approaches to support resource adequacy analysis.

Background

In this continuing effort to improve NERC’s probabilistic and deterministic assessments, the now-disbanded Probabilistic Assessment Improvement Task Force (PAITF) was formed in May 2015 from members of the Planning Committee (PC), the Reliability Assessment Subcommittee (RAS), and selected observers from industry, to identify improvement opportunities for NERC’s Long-Term Reliability Assessment (LTRA) and complementary probabilistic analysis. As a result of the PAITF recommendations, monthly reporting of LOLH and EUE were added for the 2016 Core Probabilistic Assessment (ProbA) report. The PAITF developed two reports; the NERC Probabilistic Assessment Improvement Plan report published in December 2015, over which possible recommendations by PAITF were provided based on recent LTRA key findings for NERC core and proposed coordinated special probabilistic assessment reports. The NERC Technical Guideline Document, which was published in August 2016, detailed probabilistic modeling guidelines and technical recommendations that serve as a platform for detailing probabilistic analytical enhancements to resource adequacy.

The PAITF defined five different widely used probabilistic resource adequacy statistics such as LOLE, LOLH, EUE, LOLP and LOLEV. Only LOLH and EUE have been reported in past NERC Core Probabilistic Assessment reports for all assessment areas.

Advancing further effort towards advocating probabilistic adequacy studies, NERC formed the Probabilistic Assessment Working Group (PAWG) in December 2016 with a primary function to further advance the work initiated by the Generation and Transmission Reliability Modeling Task Force (GTRPMTF) and the PAITF for improving NERC’s Core probabilistic assessments.

Given the evolving landscape of resource mix, this technical reference report focuses on identifying, defining, and evaluating more probabilistic approaches and risk metrics for ongoing analyses in order to provide further insights into resource adequacy assessment. This report explores the approaches and applications, of commonly used Reliability Risk Metrics. The foundation of the report is based on results from a NERC survey on probabilistic studies. In particular, the report presents survey results to the electricity sector on existing and future use of probabilistic studies to investigate BPS risks to reliability, results on tracking evolving emerging reliability trends. The report as well recommends applications for the electricity sector to use known reliability metrics to assess emerging issues.

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1 Probabilistic Assessment Improvement Task Force
2 NERC 2016 ProbA Report
3 Probabilistic Assessment Improvement Task Force - Technical Guideline Document
5 Probabilistic Assessment Improvement Task Force website
NERC Survey on Probabilistic Studies

In May 2017, the NERC PAWG distributed a survey on probabilistic studies to seek information on probabilistic approaches adopted by NERC Regions and Assessment Areas, Balancing Authorities (BAs) and other industry entities in North America. The RRMs, applications, and probabilistic studies used to assess emerging reliability issues discussed in this report are based on the responses received from more than 70 survey participants in North America.\(^6\)

**Survey Objectives:**

- Review the ongoing probabilistic analyses and future plans for further insights into resource adequacy assessment.
- Understand the choice of probabilistic methods, tools, and selection of acceptable reliability levels used by NERC regions and the industry at large to assess resource and transmission adequacy.
- Show the need to expand probabilistic studies to help assess emerging reliability issues that have an impact on BPS reliability.
- Explore probabilistic approaches used that provide further insights into how to best establish adequate reserve margins amidst a BPS undergoing unprecedented changes.
- Identify how members of industry define, and apply RRMs.
- Explore applications of commonly used RRMs and how each RRM can measure different aspects of a system’s reliability, such as frequency, duration and magnitude of loss of load, depending on how the metric is defined and applied.
- Provide recommendations on the usefulness of most commonly RRMs in assessing system adequacy.

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\(^6\) More survey background information, along with the Probabilistic Studying Survey form, is included in the report as Appendices A and B.
Reliability Risk Metrics

A key consideration for resource adequacy planning is determining how much generation is needed to serve the expected future load while maintaining a desired reliability level and considering uncertainty. A related issue is whether an overbuilding or under-building of generation can be alleviated by additional market transactions with neighboring systems (exports and imports). Planners need to accurately forecast the optimum level of resources. In addition to assessing the risk to reliability, planners may also consider the financial cost and environmental burden of their decisions. In turn, this drives choices of how to ensure an adequate level of supply, such as technology type, market designs, or additional market transactions with neighboring systems.

Traditional deterministic approaches have targeted the estimated future peak load plus a planning reserve margin. More robust approaches incorporate RRM that provide an estimate of the availability of the power supply using probabilistic techniques. RRMs also allow system planners to better identify future needs and tailor their decisions accordingly. This chapter discusses common probabilistic RRMs used, basic computational approaches used in RRM calculations, and some considerations on their definitions, modeling and use.

Basic Computational Approaches

Generally, the probabilistic reliability indices of a system can be evaluated using one of the following two basic approaches: Monte Carlo Simulation and Convolution. Planners of many North American BPSs conduct resource adequacy/reliability analysis for identifying installed capacity requirements associated with the static reserve needs. Static reserve needs refer to the amount of installed capacity that is required by the BPS to allow for scheduled maintenance of and random failure of resources combined with peak load growth above the expected amounts. Many BPS planners identify their installed capacity requirements using some sort of resource adequacy criterion based on two commonly used reliability indices: the Loss of Load Expectation (LOLE, in days/year) or the Loss of Load Hour (in hours/year). In addition to these two reliability indices, there are BPSs that rely on a percent of load criterion to determine their installed capacity needs.

The calculation of probabilistic reliability indices is done using either the Monte Carlo or the analytical method. The following is a brief discussion of these two approaches and the static versus short term reserve:

- **Monte Carlo Simulation**: The Monte Carlo method simulates the actual process and random behavior of the system – treated as a series of experiments. Monte Carlo simulation approaches can be categorized as "non-sequential" and "sequential". A non-sequential simulation process does not move through time chronologically or sequentially, but rather takes only the snap shot of the system state at various time. Non-sequential Monte Carlo simulation is also called state sampling approach. A sequential Monte Carlo simulation steps through the model year chronologically, recognizing the fact that the status of a piece of equipment is not independent of its status in adjacent hours. It tries to simulate the failure and repair history of system components based on their probability distributions of their state residence time. Equipment forced outages are modeled by taking the equipment out of service for contiguous hours, with the length of the outage period being determined from the equipment’s mean-time-to-repair statistics.

In both “non-sequential” and “sequential” Monte Carlo simulation, the number of artificial history replications must be established to achieve an acceptable level of statistical convergence. The degree of statistical convergence of a reliability index is measured by the standard deviation of the estimate of the reliability. Annual indices covering the period of interest are calculated as the average of the accumulated (replication) data until the variance is equal to or smaller than the selected convergence criteria. The “sequential” Monte Carlo simulation requires more input parameters and computation time than the “non-sequential” simulation. However, the sequential simulation can model issues of concern that involve time correlations, such as unit starting times or deferred unplanned outages, and can be used to calculate indices such as frequency and duration.
• **Analytical Method (Convolution):** The analytical method for computing resource adequacy indices consists of three steps: 1) the development of the load model which describes the expected system load with uncertainty representation to capture the variation of the demand associated with the weather and or economic forecast; 2) the development of the capacity model which describe the random behavior of the capacity resource outages and the energy generation of the intermittent resources; and 3) the use of probabilistic mathematics to compute the reliability indices associated with the combination of the load and the capacity models.

Mathematically, the combination of load and capacity models to compute reliability indices involves the calculation of the distribution of the difference of two random variables. If the random variables are continues the probability density function of their sum/difference can be obtained using the convolution integral. Evaluation of convolution integral is very tedious and sometimes there may not exist an analytical solution and therefore approximate methods such as the cumulant methods are used. In this situation, the process of convolution is replaced by finding the summation of the cumulants of the distributions. If the random variables are discrete, the mean values of their sum/difference can be evaluated easily using the discrete convolution method. Some efficient discrete convolution approaches have been developed, such as recursive unit addition and equivalent load approaches. The computation time for these calculations is much faster than the Monte Carlo approach.

When the resource adequacy analysis includes the interface limit between the subareas, the problem is modeled as a probabilistic flow network. It becomes a highly multidimensional problem and Monte Carlo simulation becomes more suitable. So in the multi-area reliability analysis involving transmission interfaces, the Monte Carlo approach or the hybrid Monte Carlo/Analytical Approach is usually required.

The following are definitions of commonly used RRM s that can be produced for different time intervals. Some RRM s are best suited for determining static or long term reserve needs. Short term or dynamic reserve needs are not typically identified using RRM s.

The core of evaluating system reliability is quantifying the amount of demand not served (or loss of load). Demand not served at hour i in the kth Monte Carlo iteration is defined in Equation 1 as follows:

$$\text{DNS}_{k} = \max\{0, L_i - \sum_{j=1}^{m} G_{jk}\}$$  

(1)

Where $L_i$ is the load in hour $i$, $G_{jk}$ is the available capacity of the $j$th generator in the $k$th sampling (Monte Carlo iteration), and $m$ is the number of generators in the system. DNS$_{ki}$ is the amount of demand not supplied in hour $i$, in the $k$th iteration (in MW).

$I_{ki}$ is a Boolean variable representing whether there is demand not supplied in hour $i$, in the kth iteration using the following definition:

$$I_{ki}(\text{DNS}_{ki}) = \begin{cases} 
0 & \text{if } \text{DNS}_{ki} = 0 \\
1 & \text{if } \text{DNS}_{ki} \neq 0 
\end{cases}$$

(2)

Below are definitions of the common RRM used in industry for reliability assessments.

---

7 Dynamic or short term reserve is a reserve requirement that changes according to the size of the largest contingency or the two largest contingencies the operator is trying to protect.
Loss of Load Hours (LOLH)

Definition
Loss of Load Hours (LOLH) is generally defined as the expected number of hours per time period (often one year) when a system’s hourly demand is projected to exceed the generating capacity. This metric is calculated using each hourly load in the given period (or the load duration curve).

Methods for Calculation-Computation Methods
LOLH is calculated in two steps:
1. Count the number of hours where there is loss of load in each iteration, Equation 3.
2. Average the number of hours (from step 1) across all iterations, Equation 4.

Monte-Carlo
These two steps are shown in the equation below

\[ L_{LOLH}^k = \sum_{i=1}^{H} I_{ki} \]  

Where \( L_{LOLH}^k \) is the loss of load duration (in hours) in the \( k \)th iteration, \( i \) is a variable representing each hour, \( H \) is the total number of hours in the study period such as 8760, and \( I_{ki} \) is a Boolean variable representing whether there is demand not supplied in hour \( i \), in the \( k \)th iteration. LOLH can be then calculated as shown in equation (4):

\[ LOLH = \frac{1}{N} \sum_{k=1}^{N} L_{LOLH}^k \]  

Where \( k \) is an index representing an iteration, and \( N \) is the total number of iterations.

Analytical
The analytical method used to determine the hourly loss of load probability for each hour \( i \) of the study period can be described by the following formula:

\[ LOLH = \sum_{i=1}^{H} LOLP_i \]  

Where \( i \) is variable representing each hour, \( H \) is the total number of hours in the study period such as 8760, and \( LOLP_i \) is the Loss of Load Probability in hour \( i \). Equation (5) is also valid in a Monte-Carlo context, provided that \( LOLP_i = \frac{1}{N} \sum_{k=1}^{N} I_{ki} \).

Classic analytic calculations would use monthly or annual random variable distributions for which these equations would not work\(^8\).

Considerations on the Use of LOLH
LOLH should be evaluated using all hours, rather than just peak periods. It can be evaluated over seasonal, monthly or weekly study horizons. LOLH does not inform of the magnitude or the frequency of loss of load events, but it is used as a measure of their combined duration. LOLH is applicable to both small and large systems and is relevant for assessments covering all hours (compared to only the peak demand hour of each season). LOLH provides

\(^8\) Please see Appendix C-2 for example of LOLP calculation.
insight to the impact of energy limited resources on a system’s reliability, particularly in systems with growing penetration of such resources. Examples of such energy limited resources include:

- Demand Response programs, which can be modeled as resources with specific contract limits including hours per year, days per week, and hours per day constraints,
- Energy Efficiency programs, which can be modeled as reductions to load, with an hourly load shape impact
- Distributed resources, such as behind the meter PV, which can be modeled as reductions to load, with an hourly load shape impact

**Loss of Load Events (LOLEV)**

**Definition**
Loss of Load Events (LOLEV), also known as Loss of Load Frequency (LOLF), is defined as the number of events in which system load is not served in a given time period. A LOLEV counts the expected frequency of continuous loss of load hours.

**Methods for Calculation-Computation Methods**
LOLEV is calculated on all hours, not just daily peak hours. Both Monte Carlo and Convolution methods can be used for evaluating this metric. The risk metric is evaluated using the following formula in Monte Carlo simulation:

$$LOLEV = \frac{1}{N} \sum_{k=1}^{N} LLO_k$$

(6)

Where $LLO_k$ is the total number of loss of load occurrences, $k$ is an index representing each iteration, and $N$ is the total number of iterations.

**Considerations on the Use of LOLEV**
LOLEV does not reflect magnitude or duration of loss of load but rather counts how many loss of load events occurred for a consecutive amount of hour(s) in a given time period. LOLEV is useful if considered alongside other metrics specified in this report when evaluating capacity planning decisions. For example, a system where the LOLH and LOLEV are approximately equal would indicate that most events are short in duration, more precisely LOLH and LOLEV are the average duration of outages.

LOLEV does not take into consideration the duration or magnitude of the individual involuntary load shed events. The LOLEV metric does not differentiate between events that last for one hour or for several continuous hours, nor an event where the loss of load is for one or several hundred megawatts of load. Note that this is not a probability index, but a frequency of occurrence index.

The LOLEV is also useful for systems where planners are concerned about the potential for multiple loss of load events in a single day. Other metrics, such as LOLE, cannot capture the risk associated with multiple events over the course of a given interval, typically a day. Multiple LLO events are much more likely to occur with significant addition of VER. As a result, resource planners may underestimate the potential for loss of load events.
Loss of Load Expectation (LOLE)

**Definition**
LOLE is defined as the expected number of days per time period (usually a year) for which the available generation capacity is insufficient to serve the demand at least once per day. LOLE counts the days having loss of load events, regardless of the number of consecutive or nonconsecutive loss of load hours in the day.  

**Methods for Calculation—Computation Methods**
Using a Monte-Carlo technique, the calculation equation is as shown below:

\[
\text{LOLE days/day} = \frac{1}{N} \sum_{k=1}^{N} \left( \frac{1}{D} \sum_{d=1}^{D} E_{k,d} \right) 
\]

(7.1)

\[
\text{LOLE days/period} = \frac{1}{N} \sum_{k=1}^{N} \sum_{d=1}^{D} E_{k,d} 
\]

(7.2)

Where, \( d \) is a variable representing a day, \( D \) is the total number of days, \( k \) is a variable representing an iteration, \( N \) is the total number of iterations, and \( E_{k,d} \) is a Boolean variable describing whether there was at least one hour of loss of load in the day:

\[
E_{k,d} = \begin{cases} 
0 & \text{if } \text{LOLH}_{k,d} = 0 \\
1 & \text{if } \text{LOLH}_{k,d} \neq 0 
\end{cases} \]

(8)

Where \( \text{LOLH}_{k,d} \) is the loss of load duration for a day for each iteration, shown below is the calculation equation:

\[
\text{LOLH}_{k,d} = \sum_{i=1}^{H_d} l_{ki} 
\]

(9)

Where \( i \) is a variable representing each hour, and \( l_{ki} \) is a Boolean variable representing whether there is demand not supplied in hour \( i \), in the \( k \)th iteration, and \( H_d \) is the total number of hours in a day being evaluated.

**Analytical Technique**
Using an analytical technique, the LOLE can be calculated using the equation below:

\[
\text{LOLE} = \sum_{d=1}^{D} \max_{i=1}^{H_d} (\text{LOLP}_i) 
\]

(10)

**Considerations on the Use of LOLE**
Entities could evaluate all hours of a given time period when calculating LOLE, especially when considering the impact of changing resource mix (particularly DERs and VERs) is having on the daily load distributions of many areas across the BPS. With the addition of intermittent resources, it is becoming more difficult to argue that there is not likelihood of loss of load during unmodeled hours such as off-peak or weekend hours.

Based on the Probabilistic Survey Study results, 74 percent of entities using LOLE evaluate all hours (8,760 hours/year), while 16

9 Industry experts utilize various techniques from evaluating only the daily peak hour, subset of daily hours, or all daily hours. More on this topic under the “Considerations on the Use of LOLE” section.
percent only evaluate the daily peak hours (365 hours/year). The remaining 10 percent consists of two entities, one of which excludes daily peaks on weekends and the other only evaluates the summer and winter peak hour. Also, to allow easy comparison between entities, it is recommended that entities report the time period and hours associated with their LOLE calculation and the reasoning behind their approach. For instance, the LOLE evaluated on just the daily peak hours will always be equal to or less than an LOLE based on all hours. System characteristics, such as the kurtosis (relative peakiness) of the daily load profile, hourly generator performance, and other factors, determines the magnitude of the delta between the two LOLE calculations.

This is illustrated using a generic system example, shown in Appendix B, where one iteration (#5) did not have loss of load during the peak hour. This iteration impacts the LOLE daily peak hours vs. all hours calculations. In this case the all hours LOLE of 2 is greater than the daily peak hours LOLE of 1.8.

**Loss of Load Probability (LOLP)**

**Definition**

This is defined as the probability of system daily peak or hourly demand exceeding the available generating capacity during a given period. The probability can be calculated either using only the daily peak loads (or daily peak variation curve) or all the hourly loads (or the load duration curve) in each study period.

**Methods for Calculation-Computation Methods**

A Monte-Carlo based approach is based on the mathematical process of random sampling from the generation availability and demand distributions and re-iterating the process to determine how many times there is a loss of load. The number of Loss of Load events divided by the number of possible Loss of Load events is the calculation of LOLP.

**Formula (Using Monte-Carlo Sampling):**

1. Assume $G_{jk}$ is the available capacity of the $j$th generator in the $k$th sampling, and $m$ is the number of generators in the system;

   
   \[
   \text{System Available Capacity} = \sum_{j=1}^{m} G_{jk}
   \]  

2. $L_i$ is the load at the $i$th hour;

   \[
   L_i
   \]

3. Demand Not supplied $D_{DNS_{k,i}}$ in the $k$th sampling; If Load is less than System Available Capacity this equation will equal 0.

   \[
   D_{DNS_{k,i}} = \max\{0, L_i - \sum_{j=1}^{m} G_{jk}\}
   \]  

4. If Load is greater than Generation Availability, set $I_{k,i} = 1$, otherwise 0;

   \[
   I_{k,i} = \max\{0, L_i \{0 \text{ if } D_{DNS_{k,i}} = 0 \}
   \]

5. $N$ is the number of replications; LOLP is the count of the times load is greater than availability divided by the number of samplings;

   \[
   \text{LOLP} = \frac{1}{N} \sum_{k=1}^{K} I_k
   \]
Reviewing the formulas above, it is important to note that the LOLP calculation using a Monte-Carlo approach is a count of how many test periods produce a loss of load in each sample. Therefore, the calculation is highly dependent on what periods are being analyzed.

**Considerations on the Use of LOLP**
LOLP can be calculated for any study period based on numerous time increments of the study period. Either way, the calculation is the same, the count of the periods with loss of load divided by the total number of periods in each sample.

**Expected Unserved Energy (EUE)**

**Definition**
The EUE is the summation of the expected number of megawatt hours of demand that will not be served in a given time period as a result of demand exceeding the available capacity across all hours. EUE is an energy-centric metric which considers the magnitude and duration for all hours of the time period, calculated in megawatt hours (MWh).

This measure can be normalized based on various components of an Assessment Area (i.e., total of peak demand, Net Energy for Load, etc.). Normalizing the EUE provides a measure relative to the size of a given Assessment Area. One example of calculating a Normalized EUE part per million or ppm is defined as.

\[
EUE \text{ (ppm)} = \frac{EUE \text{ (MWh)}}{\sum_{i=1}^{N} L_i} * 10^6
\]  

(15)

**Methods for Calculation-Computation Methods**
EUE can be calculated using Monte Carlo or convolution, by applying the following formula:

\[
EUE = \frac{1}{N} \sum_{k=1}^{N} ENS_k
\]  

(16)

Where \( ENS_k \) is Energy Not Supplied in \( k \)th iteration, and \( N \) is the total number of iterations.

**Considerations and Recommendations on the Use of EUE**
EUE is the only metric which considers magnitude of loss of load events. With the changing generation mix, to make EUE a more effective metric, hourly EUE should be reported for every month or year (24 data points).

**Summary of Reliability Risk Metrics**
System needs can be described using three characteristics; frequency, duration, and magnitude. As shown in the summary table 1 below, each Reliability Risk Metric allows planners to identify one or multiple of these characteristics.
<table>
<thead>
<tr>
<th>RRM</th>
<th>Frequency</th>
<th>Duration</th>
<th>Magnitude</th>
<th>Hours Considered</th>
<th>Calculation Method</th>
</tr>
</thead>
<tbody>
<tr>
<td>LOLH</td>
<td>No</td>
<td>Yes</td>
<td>No</td>
<td>All Hours</td>
<td>Monte Carlo or Convolution</td>
</tr>
<tr>
<td>LOLEV</td>
<td>Yes</td>
<td>No</td>
<td>No</td>
<td>All Hours</td>
<td>Monte Carlo or Convolution</td>
</tr>
<tr>
<td>LOLE</td>
<td>Yes</td>
<td>Yes</td>
<td>No</td>
<td>Peak Hours or All Hours</td>
<td>Monte Carlo</td>
</tr>
<tr>
<td>LOLP</td>
<td>Yes</td>
<td>Yes</td>
<td>No</td>
<td>All Hours</td>
<td>Monte Carlo or Convolution</td>
</tr>
<tr>
<td>EUE</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>All Hours</td>
<td>Monte Carlo or Convolution</td>
</tr>
</tbody>
</table>

10 Frequency is the count of the number of loss of load events over a particular period of time or in a given sample.

11 Duration is the length of time of a loss of load event.
Applications

Although common reliability metrics such as LOLE, LOLP and LOLEV have been used extensively for a long time, they are not metrics used in the NERC Core Probabilistic Assessment to avoid potential conflicts with regional practices based on different methods.

How members of industry define, and apply these reliability metrics may vary. This section sheds some light on metrics applications by the industry at large to find commonality and consistencies throughout RRM based on results from an industry survey.

LOLP
LOLP can be used to determine the probability or likelihood of event due to insufficient capacity. LOLP can be compared across studies and areas as the probability of occurrence, in between 0 and 1 producing results on a common spectrum.

EUE
Among survey responses, 20 of them calculate EUE in their probabilistic studies. EUE is widely used not only in probabilistic studies but also in other planning studies since it is an important indicator of system adequacy and easy to calculate. EUE is very useful in estimating the size of loss of load events so the planners can estimate the cost and impact of the loss of load events. EUE can be used as basis for reference reserve margin to determine capacity credits for variable energy resources. In addition, EUE can be used to quantify the impacts of extreme weather, common mode failure etc.

LOLH
As demonstrated by the results of the attached survey, the LOLH metric is computed by a large number of entities in North-America. However, only one entity, uses this metric as a reliability criterion, with their criterion set at 2.4 hours per year.

Outside of North America, this metric appears to be more widely used as a reliability criterion, particularly in Western Europe, with criteria ranging from 3 to 8 hours per year.

LOLE
The majority of entities conducting loss of load expectation studies primarily use LOLE to establish resource adequacy criteria. Criteria development Entities may also leverage other metrics and factors in their criteria development to determine a sufficient reserve margin to maintain an adequate level of system reliability. LOLE generally helps inform integrated resource planning, market-based resource procurement, generator interconnection queue projects, and other planning activities.

Some system planners may also choose to optimize their resource adequacy criteria based on other factors than LOLE, such as, but not limited to, expected unserved energy (EUE), system and societal costs, and the risk averseness of their regulating bodies and end-use customers. Consider the analogy of an individual’s determination of the appropriate driving speed; the criteria to travel down the highway. The miles per hour (mph) metric is inversely analogous to LOLE, measured at any given point in time, while the speed limit is one criteria, similar to the industry standard 0.1 days per year LOLE, which influences the driver’s ultimate decision to align
mph to an optimal driving speed. The driver may choose to drive to the posted speed limit, or may choose to optimize based on other factors such as car performance and the driving patterns of others on the highway. The drivers’ (system planners’) criterions may vary given the highways (systems) they are operating on.

**LOLEV**

The LOLEV metric is useful in systems that are concerned with the frequency of events, regardless of duration or magnitude. It is also useful for systems where events may occur multiple times in a single day, such as systems with a high load factor, indicating a flatter load shape (e.g. systems with predominately industrial load) or where the system is sensitive to forced outages from larger generators. In such cases, the LOLEV metric may better estimate system risk than the traditional LOLE metric.

Some jurisdictions do not differentiate between LOLEV and LOLE. In these cases, the resource adequacy standard is defined as, “one expected event per ten years.” Systems using this standard should be aware that this may lead to a higher level of reliability than applying the standard using the LOLE metric. In these cases, the metric is used to determine resource adequacy requirements for capacity planning purposes or for determining the planning reserve margin.
The resource mix and its delivery are transforming from large, remotely-located coal and nuclear-fired power plants, towards gas-fired, renewable energy limited, and distributed energy resources. These changes in the generation resource mix and the integration of new technologies are altering the operational characteristics of the grid and will challenge system planners and operators to maintain reliability. Failure to take into account these characteristics and capabilities can lead to insufficient capacity, energy, and Essential Reliability Services (ERS) (sometimes called “ancillary services”) to meet customer demands.

The focus of this section is three folds; first it surveys the electricity sector existing and future use of probabilistic studies to investigate BPS risks to reliability; second it tracks evolving emerging trends; third, it identifies applications for the electricity sector to use known reliability metrics to assess emerging issues.

The Use of Probabilistic Studies to Assess Emerging Issues
Several emerging key issues have the potential to increase risks to reliability that may require mitigation to maintain bulk power system reliability. These issues include:

- Resource adequacy;
- Single-fuel dependency;
- Nuclear uncertainty;
- Essential Reliability Services;
- Distributed Energy Resources;
- VER impact on reliability;
- Fuel security;
- Unit outages (nuclear generation curtailments);
- Transmission aging; and
- Transmission outages.

Previous NERC assessments showed the need to support probability-based resource adequacy assessment due to changing resource mix with significant increases in energy-limited resources, changes in off-peak demand, and other factors can have an effect on resource adequacy. As a result, NERC is incorporating more probabilistic approaches into its assessments including the development of this report. NERC PAWG examined the use of probabilistic studies in assessing emerging reliability issues and therefore, asked NERC regions and other members of the industry what emerging issues they use or may use probabilistic studies to investigate. The table below summarizes survey responses on key emerging reliability issues that probabilistic studies can be used to assess. Survey response on emerging issues echoed NERC’s key risk profiles and reliability priorities on areas of recommendations where further study, enhanced practices, and ongoing coordination with the industry are needed to ensure reliability.

---

12 2016 LTRA Assessment
13 ERO Reliability Risk Priorities Report, 2016
Table 2: Probabilistic Studies to Support Addressing Emerging Reliability Issues

<table>
<thead>
<tr>
<th>Emerging Issue</th>
<th>Details</th>
</tr>
</thead>
<tbody>
<tr>
<td>Generation Mix Changes</td>
<td>• Risks outside of peak hours (off season)</td>
</tr>
<tr>
<td></td>
<td>• Normal/extreme weather events</td>
</tr>
<tr>
<td></td>
<td>• Seasonality</td>
</tr>
<tr>
<td></td>
<td>• Replacement/Retirement</td>
</tr>
<tr>
<td></td>
<td>• Inertia</td>
</tr>
<tr>
<td>Integration of Variable Energy Resources</td>
<td>• Capacity Credit</td>
</tr>
<tr>
<td></td>
<td>• Resource Adequacy/Margin (installed capacity requirements/planning reserve margin)</td>
</tr>
<tr>
<td></td>
<td>• Ramping/Flexibility/Regulation</td>
</tr>
<tr>
<td></td>
<td>o Ancillary services</td>
</tr>
<tr>
<td></td>
<td>• Pricing/Congestion</td>
</tr>
<tr>
<td></td>
<td>• Tie line resource assessments</td>
</tr>
<tr>
<td>New Technologies</td>
<td>• Such as Energy Storage (Batteries), Electric Vehicles, Demand Response</td>
</tr>
<tr>
<td></td>
<td>• Distributed Resources</td>
</tr>
<tr>
<td></td>
<td>• Capacity Credit</td>
</tr>
<tr>
<td>Common Mode Failure</td>
<td>• Fuel Security/gas curtailment</td>
</tr>
<tr>
<td></td>
<td>• Single Points of Disruption</td>
</tr>
<tr>
<td>Transmission planning</td>
<td>• Congestion</td>
</tr>
<tr>
<td></td>
<td>• Stability studies</td>
</tr>
<tr>
<td></td>
<td>• Dynamic studies</td>
</tr>
</tbody>
</table>

The following table shows issues that can be addressed using probabilistic analysis and metrics that are not discussed in this report. PAWG recommends that NERC to delegate these issues to appropriate committees and working groups.

Table 3: Probabilistic Studies to Support Addressing Emerging Reliability Issues

<table>
<thead>
<tr>
<th>Emerging Issue</th>
<th>Details</th>
</tr>
</thead>
<tbody>
<tr>
<td>Operational Concerns</td>
<td>• Unit commitment</td>
</tr>
<tr>
<td></td>
<td>• Over-generation</td>
</tr>
<tr>
<td></td>
<td>• Dispatchability</td>
</tr>
<tr>
<td>Essential Reliability Services</td>
<td>• Capacity Credit</td>
</tr>
<tr>
<td></td>
<td>• Ramping</td>
</tr>
<tr>
<td></td>
<td>• Flexibility</td>
</tr>
<tr>
<td></td>
<td>• Regulation</td>
</tr>
<tr>
<td>Asset Evaluation</td>
<td>• Potential resource upgrades, viable replacement resources</td>
</tr>
</tbody>
</table>
Industry Application of Reliability Metrics into Emerging Issues
This section focuses on applications by the electricity sector of reliability metrics into emerging issues.

Loss of Load Probability (LOLP)
No respondents to the industry survey were contemplating moving from a reliability criterion based on an annual metric (LOLE) to a reliability criterion based on LOLH. Generally, LOLH is a more suitable metric in systems with known energy limitations, such as systems with high levels of hydro power generation.

Additionally, with the growing penetration of variable energy resources in comparison to traditional base load resources, either as load reducers or as supply, it is anticipated that hourly variations in load and supply will become less predictable. Time series models, which more accurately predict the behavior of stochastic processes such as the variations in wind speed and solar variations, may become more prevalent in probabilistic assessments. This change in modeling may in turn result in a metric such as LOLH, which captures hourly variations in system conditions, becoming increasingly meaningful in measuring the reliability of the system.

Expected Unserved Energy (EUE)
EUE along with value of loss load (VOLL) can be used to monetize the cost of loss of load to justify, prioritize or rank transmission or other capital projects. EUE can be used as basis for reference reserve margin to determine capacity credits for variable energy resources. In addition, EUE can be used to quantify the impacts of extreme weather, common mode failure etc.

Loss of Load Expectations (LOLE)
None of the respondents to the survey suggested use of LOLE for other purposes than to establish resource adequacy criteria. Most of the emerging issues surrounding a changing resource mix need answers to questions regarding energy loss, loss of load duration and frequency, as well as shifts in hourly loss of load probability from the historical peak time periods.

Loss of Load Expected Events (LOLEV)
The LOLEV metric can be applied to several emerging issues. With respect to generation mix changes, it is excellent metric for addressing risks outside of daily peak hours or shoulder seasons. It can also provide beneficial for integration studies of variable energy resources, as it recognizes that VERs can provide capacity value outside of daily peak hours. As the amount and percentage of distributed resources grow on systems, the LOLEV metric can be used for identifying adequacy shortfalls outside of the daily peak or frequency of loss of load events due to changing load shapes and shifting demands.

Loss of Load Hours (LOLH)
LOLH provides insight to the impact of energy limited resources on a system’s reliability, particularly in systems with growing penetration of such resources.
Conclusions

The NERC Probabilistic Assessment Working Group developed this technical reference report to identify, define and evaluate probabilistic metrics used in the industry to advance the work of the NERC Probabilistic Assessment Improvement Plan Report and Technical Guidelines Report. Significant changes in the resource mix, including the growing penetration of variable and behind-the-meter generation, have influenced changes on load profiles and have challenged reliability planners’ traditional methods of gauging adequate levels of supply for the bulk power system. These changes have increased the need to review these traditional, deterministic and probabilistic approaches to measuring the resource adequacy. As a result, NERC has analyzed these probabilistic approaches and created recommendations to meet these needs to assure adequate reserve margins are met and maintain reliability.

This technical reference report explored the approaches and applications, of commonly used Reliability Risk Metrics. It was found that each RRM can measure different aspects of a system’s reliability, such as frequency, duration and magnitude of loss of load, depending on how the metric is defined and applied. In addition, NERC analyzed commonalities and trends from industry on the application these probabilistic reliability metrics. Results indicated that there is a degree of variability on how similar metrics are defined and applied in gauging resource adequacy across the industry. Recommendations for changes to the application of RRMs were analyzed and discussed to improve their effectiveness.

In the face of changes affecting the bulk power system, NERC will continue to review and provide guidance on the development of probabilistic methods for assuring resource adequacy and reliability. These measures will allow better risk-informed recommendations by planners for policy makers in the face of increasing unpredictability and uncertainty of supply and demands, on the bulk power system.
Appendix A: Survey Building Blocks

The survey grouped questions into several categories or building blocks. Each building block represents an area for system planners can focus attention while developing analyses and future improvement plans, as they relate to system supply and transmission adequacy assessments. Table 1-A shows all building blocks used in the survey. Some of the building blocks are out of the scope of this report and have not been covered in this report.

<table>
<thead>
<tr>
<th>Building Block</th>
<th>Purpose</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Probabilistic Studies</strong></td>
<td>Understand the use of probabilistic studies at the regional, assessment areas’ levels for:</td>
</tr>
<tr>
<td></td>
<td>• Loss of load analysis, reserve margin targets and other studies and applications.</td>
</tr>
<tr>
<td></td>
<td>• Emanating actionable measures or adequacy requirements.</td>
</tr>
<tr>
<td></td>
<td>• The diversity of the frequency of studies performed.</td>
</tr>
<tr>
<td></td>
<td>• Identify emerging issues where probabilistic studies can be used for their investigation.</td>
</tr>
<tr>
<td><strong>Software/Algorithm</strong></td>
<td>Review and assess common software and algorithms used in North America to assess system reliability, particularly:</td>
</tr>
<tr>
<td></td>
<td>• Types of modeling complications or limitations</td>
</tr>
<tr>
<td></td>
<td>• Future plans pertaining software or model development</td>
</tr>
<tr>
<td></td>
<td>• Understand changes or improvements made over time or plans for the future to solve any limitations or model complications.</td>
</tr>
<tr>
<td><strong>Reliability Risk Metrics (RRMs)</strong></td>
<td>• Review common RRMs used such as Loss of Load Probability (LOLP), Loss of Load Expectation (LOLE), and Expected Unserved Energy (EUE) in probabilistic studies in North America.</td>
</tr>
<tr>
<td></td>
<td>• Identify how RRMs are defined and evaluated and evolved over time.</td>
</tr>
<tr>
<td></td>
<td>• Seek information regarding possible changes in RRMs or future plans to convert or add additional RRMs based on the transformation of the electric power grid.</td>
</tr>
<tr>
<td></td>
<td>• Define how reliability criteria such as one day in 10 years is derived from the RRMs and how are they applied.</td>
</tr>
<tr>
<td><strong>Variables</strong></td>
<td>Request modeling information on:</td>
</tr>
<tr>
<td></td>
<td>• How variables such as forecasted demand, wind and solar profiles, forced and maintenance outages, etc., are defined and modeled.</td>
</tr>
<tr>
<td></td>
<td>• What data collection is required to model variable parameters probabilistically?</td>
</tr>
<tr>
<td><strong>Internal and External Support</strong></td>
<td>• How transmission constraints and network topology are represented for simulation</td>
</tr>
<tr>
<td></td>
<td>• What level of external support or level of detail of external systems modeled in the probabilistic studies are needed to meet certain risk metric thresholds</td>
</tr>
</tbody>
</table>
## Demand Modeling

- How Demand-Side Management (DSM), Distributed Energy Resources (DER), Behind the Meter (BTM) are captured in the probabilistic studies
- What level of visibility required to accurately model DER and BTM in probabilistic studies?

## Reserve Margin

Review the use and purpose of a target or reference reserve margin and how they are established or calculated above demand needs.

## Criteria/Methodology

Understand how probabilistic models in North America are adjusted to meet reliability criteria if a certain risk metric threshold is not reached.
Appendix B: Survey Questions

1. Please enter the requested information below.

Region or Utility Name: [Blank]
Survey Respondent(s): [Blank]
Email and Phone Number: [Blank]
Date Survey Completed: [Blank]

2. What do you use probabilistic studies for?
Explanation: At the regional levels probabilistic studies are used for loss of load analysis while others use them for reference margin setting, etc…

☐ Planning Reserve Margin
☐ Loss of Load Expectation
☐ Ramping Capabilities
☐ Effective Load Carrying Capabilities
☐ Transmission Planning Studies
☐ Other (Please specify in your response)

3. What actionable information emanates from this analysis? What information and how it is used?
Explanation: Results from the studies can sometimes feed into actionable measures or requirements.

4. What is the frequency of the probabilistic studies? Why?
Explanation: Studies performed annually, seasonally, monthly, etc…

5. What emerging issues do you use or may use probabilistic studies to investigate?
Explanation: Emerging issues such as variable resource integration, flexible resource capabilities, etc…

6. What software is used?
Explanation: Examples like GE-MARS, SERVM, etc..

7. What solving algorithm is used?
Explanation: Examples like Monte Carlo, Convolution, etc..
8. Modeling complications?
Explanation: Any limitations or complications you have run into when trying to perform the studies. Examples like software limitations, renewable modeling time series vs. ELCC, interconnected vs islanded systems, computational runtime, market parameters, etc…

9. Changes over time?
Explanation: Have you been able to resolve the complications, if so how?

10. Future plans to change/add more software tools?
Explanation: Any future plans pertaining to software development or model changes.

11. What metrics are you using?
Explanation: What metrics are you studying in your probabilistic studies? Examples are Loss-Of-Load Probability (LOLP), Loss-Of-Load Expectation (LOLE), Expected Unserved Energy (EUE), etc…

☐ Loss-of-Load Probability (LOLP)
☐ Expected Unserved Energy (EUE)
☐ Loss-of-Load Hours (LOLH)
☐ Loss-of-Load Expectation (LOLE)
☐ Loss-of-Load Events (LOLEV)
☐ Other (please specify)

12. How are the metrics defined?
Explanation: Formulas to calculate the metrics, or what the criteria mean to you.

13. Are the metrics based on certain hours of the day? Such as peak hours vs. all hours?
Explanation: Sometimes metrics are only applied to the daily peak hour sometimes to all hours.

14. What horizon is being used (weekly, monthly, seasonal, and annual)?
Explanation: Are the metrics calculated for different time periods like an overall annual risk metric or weekly risk metrics?
15. Do different time horizons/seasons drive the use of different metrics? 
Explanation: Do you find the need to study different metrics depending on what period is being studied?

16. Any plans to change and/or add risk metrics? 
Explanation: Any future plans to convert to other metrics and why?

17. Have the metrics changed over time and why changes were made? 
Explanation: How have the metrics studied evolved over the years?

18. Do you evaluate reliability costs as part of your probabilistic studies? 
Explanation: Some areas assess the economics of reducing risk metric values. For example, this can be accomplished by accounting for incremental resource capital/production costs, Value of Lost Load (VOLL), and costs.

19. What criteria is derived from the metrics? And how are they applied? 
Explanation: For example a 1 day in 10 criteria is derived from LOLE metric.

20. What variables are modeled stochastically, and parameters varied for scenario analysis? 
Explanation: i.e., Demand, Load Forecast Uncertainty, Generator Unplanned Outages, Transmission Unplanned Outages, Variable Resource Generation, etc.

21. How are the variables identified in question 20 modeled? 
Explanation: Some areas use probabilistic distributions around an expected forecast and then randomly sample from these distributions.

22. What data is being used to model the variables identified in question 20? 
Explanation: For example, what renewable data you collect to model your variable resources? GADS data used for planned outages and maintenance.

23. Are internal transmission constraints modeled? 
Explanation: Internal transmission constraints could be modeled in probabilistic studies by using a transportation model logic or a multi-area reliability model to assess the transmission import or export constraints that would impact system or sub-area risk metrics.
24. How are transmission constraints and network topology represented for simulation?
Explanation: Examples are Nodal (all topology is modeled to the bus-level) or Zonal (All major constraints are modeled in a "Bubble & Pipe" representation

25. Is external support or demand modeled in the probabilistic studies?
Explanation: Are other areas connected to your system that might impact system or sub-area risk metrics through transmission import or export needs.

26. How much external support is relied upon in the probabilistic studies?
Explanation: Is there constant imports needed to meet certain risk metric thresholds?

27. Does your probabilistic studies capture Demand-Side Management (DSM)? If so, describe how that is accomplished.
Explanation: DR programs which are dispatchable can be modeled as energy limited resources with values for capacity and energy. EE programs which are typically non-dispatch able can be modeled as non-dispatch able resources with values for capacity and an hourly impact profile or shape.

28. Does your probabilistic studies capture Distributed Energy Resources (DER) or Behind-The-Meter (BTM) generation? If so, describe how that is accomplished.
Explanation: DR programs which are dispatchable can be modeled as energy limited resources with values for capacity and energy. EE programs which are typically non-dispatch able can be modeled as non-dispatch able resources with values for capacity and an hourly impact profile or shape.

29. If so, what level of BTM or DER visibility do you have to model such variables?
Explanation: Is there a way that you capture what may or may not have been contributed to the system by these types of variables?

30. Do you establish a target or reference reserve margin?
Explanation: Amount of capacity above demand needs for reserve purposes.

31. If so, how is the target or reference reserve margin calculated and how is the reserve margin applied to the assessment area?
Explanation: Some areas set a reference reserve margin based on a Loss-Of-Load Expectation (LOLE) of 1-day-in-10 criteria.

32. What is the purpose of setting the reference reserve margin?
Explanation: Is it set for compliance reasons, state & provincial requirements, or best practices.

33. For your modeling, how do you adjust your system to meet reliability criteria if a certain risk metric threshold is not reached?
Explanation: Examples would be adjust load or adjust resources.

34. What other types of data/details not discussed above are included in your probabilistic modeling?
Explanation: Anything not discussed above that you believe is important to note in your probabilistic studies? (Without going into specific details on modeling or results).
Appendix C-1: Reliability Risk Metrics Calculations Monte Carlo Approach

The following example applied to a generic system simplifies the calculations of RRM using simulation methods. This example shows two days of MW demand and available supply in five simulations or iterations of available supply. Table 1 shows the values in all hours of:

- Demand Not Supplied (DNS) which is the MW supply minus demand as seen in the highlighted cells of Table 1-C.
- The count of loss of load hours, a count of each hour in all five simulations where demand is exceeding supply for that hour.
- Hourly LOLP is calculated by dividing the count of loss of load in each hour by the number of simulations or iterations.

Table 2-C shows the values in each iteration of the following:

- Loss of load periods: This is a count of hours where demand exceeds supply for each iteration.
- Loss of Load Occurrences (LLO): this is a count of consecutive periods where demand exceeds supply.
- LLO Days: this is a counter of the number of days where loss of load events occur.

Table 1-C: Generic System Demand & Available Supply

<table>
<thead>
<tr>
<th>Hour (i)</th>
<th>Demand in MW (L)</th>
<th>Available Supply in MW, 5 iterations</th>
<th>Supply Minus Demand(^\dagger) in MW, 5 iterations</th>
<th>Count of LOL Hour</th>
<th>Hourly LOLP</th>
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\(^\dagger\) Highlighted cells are Demand Not Supplied (DNS)
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<td>(336)</td>
<td>(509)</td>
<td>(344)</td>
<td>(690)</td>
<td>5</td>
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</table>
Calculations of common RRM as shown in Table 3-C using demand and available supply data of the generic example are as follows:

- **LOLH**: is determined by computing the average loss of load duration over all iterations as follows:
  \[ \text{LOLH} = \frac{9 + 18 + 14 + 15 + 12}{5} = 13.6 \text{ Hours/period} \]
  Alternatively, the LOLH can be arrived at by summing the hourly LOLP over all 48 hours of the simulation:
  \[ \text{LOLH} = 0 + 0 + \cdots + 0.4 + 0.6 + 0.2 + 0.4 + \cdots + 0.8 + 0.2 + 0.2 + \cdots + 0 = 13.6 \text{ hours}. \]

- **LOLEV**: is determined by computing the average of the LOLE Occurrences (LLO) as follows:
  \[ \text{LOLEV} = \frac{3 + 5 + 4 + 4 + 7}{5} \text{ Events/period} \]

- **LOLE Days/day**: is determined by averaging the summation of the peak hour loss of load probability for each day as follows:
  \[ \text{LOLE} = \frac{0.8 + 1}{2} = 0.9 \text{ Day/day} \]

<table>
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<th>Table 2-C: Loss of Load Statistics</th>
<th>Available Supply (G)</th>
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<tr>
<td>Iteration #</td>
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<tr>
<td>Count of LOL Periods</td>
<td>9</td>
</tr>
<tr>
<td>Count of LOLE Occurrences (LLO)</td>
<td>3</td>
</tr>
<tr>
<td>Number of day where least a single LLO occurs</td>
<td>2</td>
</tr>
</tbody>
</table>
• **LOLE days/period**: is determined by the summation of the peak hour loss of load probability for each day as follows:

\[
\text{LOLE} = 0.8 + 1 = 1.8 \text{ Days/period}
\]

• **LOLP**: is determined by dividing the summation of the Loss of load periods by the total number of iterations multiplied by the number of hours as follows:

\[
LOLP = \frac{9 + 18 + 14 + 15 + 12}{5 \times 48} \times 100(\%) = 28.33\%
\]

• **EUE**: is determined using equation (16) as the summation of the amount of demand not supplied in all hour for all iterations divided by the total number of iterations.

\[
EUE = \frac{(9,644 + 11,060 + 12,103 + 10,149 + 8,250)}{5} = 10,241
\]

<table>
<thead>
<tr>
<th>Table 3-C: Calculations of Reliability Risk Metrics (RRM)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>LOLH (Hours/ period)</strong></td>
</tr>
<tr>
<td><strong>LOLEV (Events/ period)</strong></td>
</tr>
<tr>
<td><strong>LOLE (Days/ day)</strong></td>
</tr>
<tr>
<td><strong>LOLE (Days/ period)</strong></td>
</tr>
<tr>
<td><strong>LOLP (%)</strong></td>
</tr>
<tr>
<td><strong>EUE (MWh)</strong></td>
</tr>
</tbody>
</table>

Figure 1-C shows second iteration supply, demand, and loss of load events in day one and day two. Using iteration two, three loss of load events in day 1 are found whereas 2 loss of load events in day 2.

**Figure 1-C. Generic System Loss of Load Events –Iteration 2**
Appendix C-2: Reliability Risk Metrics Calculations – Analytical Approach

The examples in this Appendix show the calculations of LOLP index using analytical methods based on given load profile and generation unit Forced Outage Rate (FOR).

This Appendix covers two analytical methods based on discrete marginal density: the conventional analytical method and the Equivalent Load Method.

**Conventional Analytical Method**

The conventional analytical method for computing resource adequacy indices consists of three steps:

1) The development of the load model which describes the expected system load with uncertainty representation to capture the variation of the demand associated with the weather;

2) The development of the capacity model which describe the random behavior of the capacity resource outages and the energy generation of the intermittent resources; and

3) The use of probabilistic mathematics to compute the reliability indices associated with the combination of the load and the capacity models.

The third step is a convolution procedure. For a large system, most of the computation time is used at the third step if the number of load levels is large.

**Equivalent Load Method**

Another method, known as equivalent load method, simplified the three steps into two steps:

1) compute a suitable load model.

2) Modify the load model directly by the parameters of each unit.

After the load model is modified by all units. The probability (and frequency) for each load level is obtained and each load level can be interpreted as a margin so that the probability and frequency of each margin can be directly computed.

The equivalent load method is especially useful for Demand Response (DR) resource. One can estimate the expected number of DR resource required at certain margin at which the DR is called on. One can also estimate how long the DR will be required in each event.

A summary of the analytical approach for computing resource adequacy indices in form of conventional and equivalent load methods is provided below. Numerical examples are presented to illustrate the calculation of the LOLP and marginal distribution.

**Derivation of required calculation formula**

The random variable in the analysis of generation system reliability is the capacity Y with a given probability distribution. Two functions of this random variable (r.v.) are of interest, the cumulative probability, \( \Pr(Y < y) \), and the cumulative frequency, \( \text{Fr}(Y < y) \), in which y is a discrete capacity level.

A more useful r.v. for the purposes of analysis is the capacity loss X of a unit, which is defined by:

\[
X = Z - Y
\]
Where: Z is rated capacity of the unit. The corresponding functions for capacity loss are defined as $Pr(X>x)$ and $Fr(X>x)$. Where $x$ is given by:

$$x = Z - y$$  \hspace{1cm} (2)

The relationship of the probability and frequency represented by $X$ and $Y$ can be expressed as follows:

$$Pr(Y \leq y) = Pr(Z - X \leq Z - x) = Pr(X \geq x) \hspace{1cm} (3)$$

$$Fr(Y \leq y) = Fr(Z - X \leq Z - x) = Fr(X \geq x) \hspace{1cm} (4)$$

In the discrete r.v. case the probability is:

$$Pr(X \geq x_i) = Pr(Y \leq y_i) = \sum_{k \geq i} p(x_k) \hspace{1cm} (5)$$

Where $$p(x_k) = P(x_k) - P(x_{k+1}) = p_k$$ \hspace{1cm} (6)

and $$P(x_k) = Pr(X > x_k)$$

The probability $P(x_0)$ is usually known as cumulative probability while $p_k$ is known as an exact state probability or probability density. Thus the reliability characteristics of a power system element can be described by a variable $x_i$ the outage capacity. It takes on discrete values $x_i (i = 0, 1 \ldots N)$. $x_0$ indicating no outage and $x_{N-1}$ indicating total element failure. Of more interest are the cumulative probability and the cumulative frequency, $P(x_i)$ and $F(x_i)$, in (5) and (7).

$$F(x_i) = Fr(Y \leq y_i) = Fr(X \geq x_i) = \sum_{k \geq i} f(x_i) \hspace{1cm} (7)$$

where $$f(x_k) = F(x_k) - F(x_{k+1}) \hspace{1cm} (8)$$

Suppose that C is an element produced by the parallel connection of two elements A and B in a power system. The exact probability is:

$$p_c(x_k) = \sum_{x_i + x_j = x_k} p_a(x_i)p_b(x_j)$$

$$= \sum_{j=0}^{N_b-1} p_a(x_k - x_j)p_b(x_j) \hspace{1cm} (9)$$

Where $p_a(.)$, $p_b(.)$ and $p_c(.)$ are the exact state probabilities of elements A, B and C respectively and $N_b$ is the number of states in element B.

The cumulative probability is then given by Eq.(5):
Expressions equivalent to (9) and (10) are:

\[ P_c(x_k) = \sum_{i=0}^{N_a-1} p_a(x_i) p_b(x_k - x_i) \]

\[ P_c(x_k) = \sum_{i=0}^{N_a-1} (P_a(x_i) - P_a(x_{i+1})) P_b(x_k - x_i) \]  

(11)

For frequency function one obtains:

\[ f_c(x_k) = \sum_{x_i + x_j = x_k} |p_a(x_i) f_b(x_j) - f_a(x_i) p_b(x_j)| \]

\[ = \sum_{j=0}^{N_b-1} |p_a(x_k - x_j) f_b(x_j) + f_a(x_k - x_j) p_b(x_j)| \]

(12)

The cumulative frequency is:

\[ F_c(x_k) = \sum_{x_m \geq x_k} f_c(x_m) \]

\[ = \sum_{j=0}^{N_b-1} \sum_{x_m \geq x_k} |p_a(x_m - x_j) f_b(x_j)| \]

\[ + f_a(x_m - x_j) p_b(x_j) \]

\[ = \sum_{j=0}^{N_b-1} \left[ P_a(x_k - x_j) f_b(x_j) \right] + F_a(x_k - x_j) p_b(x_j) \]

(13)

If \( f_b \), \( p_b \) are represented by cumulative probability and frequency then:
The reliability characteristics of power system elements are thus described by the probability and frequency associated with the r.v. X outage capacity. A tabulation of this probability and frequency functions for discrete values of X is called the generation system model. At present, the conventional approach using discrete distribution method to calculate frequency and duration indices for a given power system proceeds in three basic steps:

1. Develop a suitable capacity model from the parameters of the individual generating units.
2. Develop a suitable load model from the given data over the period of study.
3. Combine the capacity model with the load model to obtain a probabilistic model of system capacity adequacy.

The load model used in the LOLP method is usually the cumulative curve of daily peak loads. Most electric power utilities can provide load data such as peak load and daily or hourly load curve. The load model is developed by a single scan of the hourly load data and has the following form:

\[ P(L_i) \cdot F(L_i) = \text{Probability and frequency of load equal to or greater than } L_i, \]

Where: \( L_{i+1} - L_i = Z \), \( Z \) is constant

One cannot predict the reasonable number of load levels for a particular system and load before computation. A small value of \( Z \) is therefore chosen, making the number of load levels large. The result is that a convolution of two large models will appear in the third step. From the analysis of number of operations, we shall see that most of the computation time is used at the third step.

In the third step, the capacity model and load model are combined to yield the probability and frequency of the margin states. Margin is defined to be the available capacity minus load and a cumulative margin state contains all states with margin less than or equal to the specified margin.

Similar to (9) and (10) one obtains:

\[ F_c(x_k) = \sum_{j=0}^{N-1} \left[ P_a(x_k - x_j)(F_h(x_j) - F_h(x_{j-1})) \right] \]

\[ + P_h(x_k - x_j)(F_a(x_j) - F_a(x_{j-1})) \]

(14)

If the summation is over the states of system A, an equivalent expression is:

\[ F_c(x_k) = \sum_{j=0}^{N-1} \left[ F_h(x_k - x_j)(P_a(x_j) - P_a(x_{j+1})) \right] + P_h(x_k - x_j)(F_a(x_j) - F_a(x_{j+1})) \]

(15)
Where: \( p_g(.) \), \( p_l(.) \) are the exact state probabilities in generation model and load model respectively. Similarly \( P_g(.) \), \( P_l(.) \) are cumulative probabilities in generation and load model respectively.

Similar to (12) and (13) one obtains:

\[
p(M) = \sum_{C \leq x_i - L_i = M} \frac{p_g(x_i) p_l(L_i)}{p_l(C - x_i - M)} = \sum_{i=0}^{N-1} p_g(x_i) p_l(C - x_i - M)
\]

\[
P(M) = \sum_{m \leq M} \sum_{i=0}^{N-1} p_g(x_i) p_l(C - x_i - m) = \sum_{i=0}^{N-1} (P_g(x_i) - P_g(x_{i+1})) P_l(C - x_i - M)
\]

Where: \( p(M), f(M) \) -- Probability and incremental frequency of margin \( M \).
\( P(M), F(M) \) = Probability and frequency, of margin less than or equal to \( M \).
\( Fg(M), Fl(AM) \) = Components of \( F(M) \) due to generation and load change respectively.
\( C \) = Installed capacity minus capacity on planned outage.
\( x \) = Capacity outage in state \( I \), \( x_0 = 0 \) and \( X_{N-1} = C \).
\( N \) = Number of capacity outage states.

The above is a derivation of the conventional analytical method. The following is a description of the equivalent Load method.

**Equivalent load approach**
In equivalent load method each unit model is viewed as a load model with state capacities represented by negative load values and combined with load model. The probability and frequency are computed for each load level \( L \). At the end an equivalent load model is obtained. The reliability indices associated with each load level in the equivalent load model are equal to the indices of corresponding negative margin in the conventional method. Denote \( -M \) by an equivalent load \( L_e \), i.e. \( L_e = -M \) and suppose \( C_i = C - x_i = \text{capacity of a unit or subsystem in state } i \). then we have:

\[
P(L^e_k) = P_f(L^e_k \geq L^e_k) = \sum_{i=0}^{N-1} p_g(x_i) P_i(L^e_k - C_i)
\]

Also from (19) one gets:

\[
F(L^e_k) = F_i(L^e_k) - F_g(L^e_k) = \sum_{i=1}^{N-1} [P_g(x_i) - P_g(x_{i+1})] F_i(L^e_k - C_i)
\]

\[
+ \sum_{i=1}^{N-1} [F_g(x_i) - F_g(x_{i+1})] P_i(L^e_k - C_i)
\]

Numerical Example

The methodology is explained by a simple example system. Only the LOLP calculation is presented. For frequency and duration the reader can find in the related reference.

The load and capacity for the 10 hour-example are shown below:

<table>
<thead>
<tr>
<th>Load Data and Load Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hour</td>
</tr>
<tr>
<td>MW</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.5</td>
<td>1</td>
</tr>
<tr>
<td>50</td>
<td>0</td>
<td>0.5</td>
</tr>
<tr>
<td>100</td>
<td>0.3</td>
<td>0.5</td>
</tr>
<tr>
<td>150</td>
<td>0.2</td>
<td>0.2</td>
</tr>
</tbody>
</table>
**Unit Parameters and Capacity Model**

<table>
<thead>
<tr>
<th>Unit Index</th>
<th>Unit Capacity</th>
<th>Failure Rate</th>
<th>Repair Rate</th>
<th>Forced Outage Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>50</td>
<td>0.1</td>
<td>0.9</td>
<td>0.1</td>
</tr>
<tr>
<td>2</td>
<td>50</td>
<td>0.1</td>
<td>0.9</td>
<td>0.1</td>
</tr>
<tr>
<td>3</td>
<td>50</td>
<td>0.1</td>
<td>0.9</td>
<td>0.1</td>
</tr>
</tbody>
</table>

**Capacity outage probability table of this system**

<table>
<thead>
<tr>
<th>Capacity In</th>
<th>Capacity outage</th>
<th>Probability</th>
<th>Cumulative Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>150</td>
<td>0</td>
<td>0.729</td>
<td>1</td>
</tr>
<tr>
<td>100</td>
<td>50</td>
<td>0.243</td>
<td>0.271</td>
</tr>
<tr>
<td>50</td>
<td>100</td>
<td>0.027</td>
<td>0.028</td>
</tr>
<tr>
<td>0</td>
<td>150</td>
<td>0.001</td>
<td>0.001</td>
</tr>
</tbody>
</table>

**System Margin**

The density function of system available capacity $C(Y)$ where $Y$ is a random variable representing the system available capacity in MW. If we define generation margin as the amount by which the system available capacity exceed the system peak load on any day, then

$$(\text{Generation}) \text{ Margin} = \text{Available Capacity} - (\text{Daily peak}) \text{ Load}$$

The Generation margin is also a random variable, $Z$, in MW with the relationship

$$Z = Y - X$$

The probability density of system margin, $M(Z)$, must be determined from the densities of load and capacity.

The System Margin can be a discrete or continuous random variable.

One can discretize the load density by dividing it into a number of discrete intervals. This results in a discrete approximation to the continuous margin density. The accuracy of the discrete margin approximation is improved by selecting a large number of intervals of small MW size.

**Discrete System Margin Density**

A full Binomial Capacity model is used for illustration.

Since the load and available capacity random variables are independent, the Margin density is given by the following relationship.

$$M(Z) = \sum_{Y} C(Y)L(Y - Z)$$

Where for each value of $Z$ the summation is taken over all values of available capacity, $Y$. 

As described in previous equations, reliability indices computation is performed by combining the capacity model with the load model to obtain a probabilistic model of system capacity adequacy.

**Using Conventional Analytical Method**

We can get the marginal distribution based on the load and capacity distribution previously calculated. This is the marginal state matrix.

<table>
<thead>
<tr>
<th>Margin (MW)</th>
<th>(Cap in , Load) Pair</th>
<th>Prob.</th>
<th>Cumulative Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>150</td>
<td>(150, 0)</td>
<td>0.3645</td>
<td>1</td>
</tr>
<tr>
<td>100</td>
<td>(100, 0)</td>
<td>0.1215</td>
<td>0.6355</td>
</tr>
<tr>
<td>50</td>
<td>(50, 0), (150, 100)</td>
<td>0.2322</td>
<td>0.5140</td>
</tr>
<tr>
<td>0</td>
<td>(0, 0), (100, 100), (150, 150)</td>
<td>0.2192</td>
<td>0.2818</td>
</tr>
<tr>
<td>-50</td>
<td>(50, 100), (100, 150)</td>
<td>0.0567</td>
<td>0.0626</td>
</tr>
<tr>
<td>-100</td>
<td>(0, 100), (50, 150)</td>
<td>0.0057</td>
<td>0.0059</td>
</tr>
<tr>
<td>-150</td>
<td>(0, 150)</td>
<td>0.0002</td>
<td>0.0002</td>
</tr>
</tbody>
</table>

The marginal distribution is calculated as:

<table>
<thead>
<tr>
<th>Load 0 MW</th>
<th>Load 100 MW</th>
<th>Load 150 MW</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>0.3945</td>
<td>0.1458</td>
</tr>
<tr>
<td>0.3</td>
<td>0.1215</td>
<td>0.0486</td>
</tr>
<tr>
<td>0.2</td>
<td>0.03645</td>
<td>0.0135</td>
</tr>
<tr>
<td>0.001</td>
<td>0.0005</td>
<td>0.0002</td>
</tr>
</tbody>
</table>

**Using Equivalent Load Method**

In a large system, the number of states of both load model and capacity model usually are large. So the marginal state matrix is a large two-dimensional matrix.

In the equivalent load method, we can avoid to calculate the each element of this large two-dimensional matrix. Instead the marginal distribution is calculated by convolving one unit capacity mode at a time. This successive updating of marginal distribution started with the original load distribution. The right three columns in the following Table show the marginal distribution after adding each unit.
The equivalent load method presented has been used to study the IEEE-RTS96.

Portion of Reliability Indices for IEEE-RTS96 are shown below:

<table>
<thead>
<tr>
<th>Equivalent Load (MW)</th>
<th>probability</th>
<th>frequency (per hr.)</th>
<th>duration (hrs.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.00</td>
<td>0.00106914</td>
<td>0.00022992</td>
<td>4.64998849</td>
</tr>
<tr>
<td>2.00</td>
<td>0.00106367</td>
<td>0.00022878</td>
<td>4.64938476</td>
</tr>
<tr>
<td>3.00</td>
<td>0.00105963</td>
<td>0.00022751</td>
<td>4.64327569</td>
</tr>
<tr>
<td>4.00</td>
<td>0.00104800</td>
<td>0.00022552</td>
<td>4.64713745</td>
</tr>
<tr>
<td>5.00</td>
<td>0.00104097</td>
<td>0.00022441</td>
<td>4.65875313</td>
</tr>
<tr>
<td>6.00</td>
<td>0.00103330</td>
<td>0.00022135</td>
<td>4.65175877</td>
</tr>
<tr>
<td>7.00</td>
<td>0.00102773</td>
<td>0.00021997</td>
<td>4.64403458</td>
</tr>
<tr>
<td>8.00</td>
<td>0.00102157</td>
<td>0.00021873</td>
<td>4.64110731</td>
</tr>
<tr>
<td>9.00</td>
<td>0.00101616</td>
<td>0.00021797</td>
<td>4.63168773</td>
</tr>
<tr>
<td>10.0</td>
<td>0.00100956</td>
<td>0.00020472</td>
<td>4.58176546</td>
</tr>
<tr>
<td>20.0</td>
<td>0.00086405</td>
<td>0.00018784</td>
<td>4.59994470</td>
</tr>
<tr>
<td>30.0</td>
<td>0.00081997</td>
<td>0.00017902</td>
<td>4.53013230</td>
</tr>
<tr>
<td>50.0</td>
<td>0.00073491</td>
<td>0.00016496</td>
<td>4.50456732</td>
</tr>
<tr>
<td>100.0</td>
<td>0.00050386</td>
<td>0.00011530</td>
<td>4.36986093</td>
</tr>
<tr>
<td>200.0</td>
<td>0.0002141</td>
<td>0.00005332</td>
<td>4.15234897</td>
</tr>
<tr>
<td>300.0</td>
<td>0.00009082</td>
<td>0.00002311</td>
<td>3.93006622</td>
</tr>
<tr>
<td>400.0</td>
<td>0.00003570</td>
<td>0.00000970</td>
<td>3.67903287</td>
</tr>
<tr>
<td>500.0</td>
<td>0.00001287</td>
<td>0.00000374</td>
<td>3.44211387</td>
</tr>
</tbody>
</table>

Appendix D: Additional Resource Adequacy Metric

Conditional Value at Risk (CVaR)
Conditional Value at Risk (CVaR) measures the expected (weighted average) outcome of tail-end events. It is commonly used in the financial sector to minimize economic risk when developing investment portfolios. For power system applications, CVaRα is used to assess the expected value of the α-percent worst outcomes. For example, the CVaR95 metric measures the expected curtailment magnitude over the worst 5-percent of potential outcomes. It has a very desirable mathematical property, namely that it is coherent, which means that it satisfies the properties of monotonicity, sub-additivity, homogeneity, and translational invariance. As a continuous function it is readily incorporated into convex and linear programing optimization models, e.g. the objective function minimizes CVaR (risk).

CVaR is currently used by Power Systems Research, Inc. in Brazil. The Northwest Power and Conservation Council also uses this variable as a cost risk measure in its system expansion model. Viable system expansion plans for the Pacific Northwest are those with the lowest expected cost over the 10 percent highest cost years (CVaR90). The Council is currently considering using CVaR as a potential adequacy metric.

Definition
Conditional Value at Risk (CVaRα), as a power supply adequacy metric, is the probability-weighted average value of the lowest surplus (capacity minus load) over the (100 – α) percentile of possible outcomes (where α typically ranges from 90 to 99). Generally, it is defined as the conditional expected surplus of all outcomes less than or equal to the Value at Risk (VaRα), which is defined as the surplus value at the (100 – α) percentile.

CVaR can be assessed using either Monte-Carlo or Probability-Convolution methods, depending on which is more appropriate.

Simple Numeric Example
Suppose a Monte-Carlo simulation produces the following 100 sorted results: {-100, -99, -98… -1}, with each result being equally likely. For an α-threshold of 95 percent, the results of interest are the lowest 5 percent, namely {-100, -99, -98, -97, -96}. For this α-threshold, there is a 5 percent chance that a result will be equal to or less than -96, thus the corresponding VaR95 value is -96. CVaR95 then is the average of all of the values from this distribution that are equal to or less than VaR95. Thus, for this example, CVaR95 = (-100 – 99 – 98 – 97 – 96) / 5 = – 98.

CVaR Calculation for a Monte-Carlo approach
For a Monte-Carlo approach, the lowest peak-hour surplus from each simulation is recorded. These surplus values are then sorted from lowest to highest. CVaRα is the average of the surplus values that are less than or equal to VaRα, that is, those that fall into the (100 – α) percentile. Therefore,

---

https://icsp2016.sciencesconf.org/file/251300
17 Seventh Northwest Conservation and Electric Power Plan, February 10, 2016,
https://www.nwcouncil.org/media/7149906/7thplanfinal_appdixl_rpm.pdf
where:
\[ \alpha = \text{Percentage used to define the values at risk, e.g. those in the } (100 - \alpha) \text{ percentile,} \]
\[ Si = \text{Surplus (capacity minus load) for the } i\text{th simulation,} \]
\[ N = \text{Number of Simulations,} \]
\[ N\alpha = \text{Simulation number of the } (100 - \alpha) \text{ percentile of } N. \]

Table 1-D below shows sorted surplus values from a created 100-simulation data set that includes the NERC sample data (containing only 5 simulations). The \( \text{VaR}_{95} \) is the surplus value for the 5\(^{th} \) percentile of the sorted simulations. For this example \( N\alpha \) is 5. Thus, \( \text{VaR}_{95} \) is \(-2458 \) megawatts and \( \text{CVaR}_{95} \) is \(-2528 \) megawatts (the average surplus of simulations 1 through 5). Figure 1-D illustrates \( \text{VaR}_{95} \) and \( \text{CVaR}_{95} \) graphically.

Table 1-D – Surplus Outcomes for a Monte-Carlo Simulation

<table>
<thead>
<tr>
<th>Simulation</th>
<th>Surplus</th>
<th>VaR(_{95})</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-2598</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>-2563</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>-2528</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>-2493</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>-2458</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>-2423</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>-2388</td>
<td></td>
</tr>
<tr>
<td>13</td>
<td>-2166</td>
<td>NERC Data</td>
</tr>
<tr>
<td>16</td>
<td>-2066</td>
<td>NERC Data</td>
</tr>
<tr>
<td>28</td>
<td>-1660</td>
<td>NERC Data</td>
</tr>
<tr>
<td>32</td>
<td>-1522</td>
<td>NERC Data</td>
</tr>
<tr>
<td>37</td>
<td>-1344</td>
<td>NERC Data</td>
</tr>
<tr>
<td>99</td>
<td>843</td>
<td></td>
</tr>
<tr>
<td>100</td>
<td>878</td>
<td></td>
</tr>
</tbody>
</table>
CVaR Calculation for a Probability-Convolution Approach

For a Probability-Convolution approach, CVaR$_\alpha$ is calculated as the probability-weighted average surplus for those values equal to or less than VaR$_\alpha$ (see Figure 2-D below for an example of $\alpha = 95$ percent).

The general expression for CVaR using a Probability-Convolution method is provided below. This method works well if the probability density distribution for surplus is well defined. CVaR$_\alpha$ is the definite integral evaluated from minus infinity to VaR$_\alpha$ of the surplus $S$ times the probability density function $P(S)$, divided by the definite integral of the probability density function over the same limits.

\[
CVaR_\alpha = E[S | S \leq VaR_\alpha] = \frac{\int_{-\infty}^{VaR_\alpha} S \cdot P(S) dS}{Pr[S \leq VaR_\alpha]} = \frac{1}{1-(\alpha / 100)} \int_{-\infty}^{VaR_\alpha} S \cdot P(S) dS
\]

where:
- $S =$ Surplus (capacity minus load),
- $P(S) =$ Probability density distribution for surplus,
- VaR$_\alpha =$ Surplus for which the area under the probability curve from minus infinity to VaR is $\alpha$. 

Figure 2-D: VaR$_{95}$ and CVaR$_{95}$ for a Probability Density Distribution

![Figure 2-D: VaR$_{95}$ and CVaR$_{95}$ for a Probability Density Distribution](image)
From the load and generation sample data provided by NERC, the highest hourly deficit for each day is selected from the 5-day hourly surplus/deficit data. The collected daily data are sorted as follows (-2166, -2056, -1660, -1522, -1344) and their populations are calculated using a 500-MW bin, which are listed in Table 2-D.

<table>
<thead>
<tr>
<th>Left Bin Boundary (MW)</th>
<th>Right Bin Boundary (MW)</th>
<th>Population</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2600</td>
<td>-2100</td>
<td>1</td>
</tr>
<tr>
<td>-2100</td>
<td>-1600</td>
<td>2</td>
</tr>
<tr>
<td>-1600</td>
<td>-1100</td>
<td>2</td>
</tr>
</tbody>
</table>

However, CVaR analysis requires more than 3 data points and ideally should include a few hundred data points. Therefore, a Gaussian function has been derived

\[ f(s) = A \exp \left[ -\frac{(s - s_{bar})^2}{2\sigma^2} \right] , \]

where \( A = 2.2 \), \( s_{bar} = -1350 \) and \( \sigma = 570 \) to closely fit these 3 points and provide enough data to properly calculate CVaR. The Gaussian function and the 3 data points are plotted in Figure 3-D below.

![Figure 3-D: Gaussian Function Fit (red curve) to the NERC Surplus Data Bins (blue squares)](image)

To enable further CVaR analysis, the Gaussian \( f(s) \) is transformed into a probability density \( p(s) \) by normalization,

\[ p(s) = \frac{f(s)}{\int_{-\infty}^{\infty} f(s)ds} = \frac{1}{\sqrt{2\pi}\sigma} \exp \left[ -\frac{(s - s_{bar})^2}{2\sigma^2} \right] , \]

which by design results in \( p(s) \) being just the Normal Distribution and is plotted in Figure 4-D below.

![Figure 4-D: Probability density function of surplus/deficit](image)
The value of VaR95 is defined to be the value s at which the probability density has accumulated the lowest 5% of the population or equivalently the value s at which the left area under p(s) has reached (1 - 0.95) = 0.05:

\[ \int_{-\infty}^{VaR_{95}} p(s)ds = \int_{-\infty}^{VaR_{95}} \frac{1}{\sqrt{2\pi\sigma^2}} \exp \left[ -\frac{(s - \bar{s})^2}{2\sigma^2} \right] ds = 0.05. \quad (1) \]

The integral in Equation (1) is evaluated iteratively via a Riemann sum approximation:

\[ \int_{-\infty}^{VaR_{95}} p(s)ds \approx \sum_{i=1}^{N} p(s_i)\Delta s = 0.05, \quad (2) \]

where \( VaR_{95} = s_M \). The Riemann sum in Equation 2 is evaluated using the sequence \((s_1...s_{28}) = (-3700...-2350)\) where \( \Delta s = 50 \) and \((s_{29}...s_{32}) = (-2340...-2310)\) with a smaller interval \( \Delta s = 10 \) to obtain a more precise determination of \( s_{32} = -2,310 \), at which the sum is approximately 0.05. The first member of the sequence \( s_1 = -3700 \) was chosen due to the fact that \( p(s_1) = 1.43 \times 10^{-7} \) was small enough to be a good representation of minus infinity where \( p(s) = 0 \). Hence, since

\[ \sum_{i=1}^{32} p(s_i)\Delta s \approx 0.05 \]

then \( VaR_{95} = 0.05 \), then \( Var_{95} \approx s_{32} = -2310 \). Finally, CVaR95 is just the \( p(s) \)-weighted average of \( s \) in the interval from minus infinity to \( VaR_{95} \) and is calculated as

\[ CVaR_{95} = \frac{\int_{-\infty}^{VaR_{95}} sp(s)ds}{\int_{-\infty}^{VaR_{95}} p(s)ds} \approx \frac{\int_{-\infty}^{-2310} sp(s)ds}{0.05} \approx -2528. \quad (3) \]