State Estimation Advancements Enabled by Synchrophasor Technology

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Executive Summary

Major power system outages, such as the August 2003 U.S.-Canada blackout, have demonstrated the need for more efficient, accurate, and reliable real-time monitoring of the power system. The major tool that is presently used to achieve this functionality in Energy Management Systems (EMS) is State Estimation (SE). Legacy State Estimators have several biases. The introduction and the continuously growing installations of Phasor Measurement Units (PMUs), which provide high resolution synchronized measurements, have opened up the possibility for more efficient and accurate monitoring of the power system. With Synchrophasor Technology SE can be reformulated both algorithmic and architectural and can contribute to eliminate the biases of present legacy SE.

The goal of this Section is to describe the advancements in SE that Synchrophasor Technology enables. Initially a brief overview of SE is provided along with the biases of legacy SE. Then it is described how Synchrophasor Technology contributes to eliminate those biases. Several formulations of synchrophasor-based SE are described i.e. hybrid, linear, distributed, three-phase and dynamic, along with their associated advantages and challenges.

State Estimation

State Estimation (SE) is the main function of an EMS. It is used for monitoring the operating condition of the system by computing a statistical estimate of the system operating state expressed through the voltage magnitude and phase of system buses and other derived quantities such as real and reactive power flows and injections. SE provides the input model for several EMS functions for both network security and market management applications such as Dynamic Security Assessment (DSA), Real Time Contingency Analysis. A flowchart of the SE function is shown in Figure 1.

![Figure 1: State Estimation Flowchart](image)

The Topology Processor is using the input status data such as circuit breaker status, interrupt switch status, etc. and provides the network model configuration for the state estimator in a bus/branch format. The Observability Analysis is defining the observable islands within the system. State Estimator is executed using the available analog measurements and pseudo-measurements (e.g. load forecasts), along with information from the Topology Processor and Observability Analysis. Bad Data Analysis is performed on the output results of the State Estimator [1].
Legacy State Estimation Biases

State Estimation at its present implementation has the following characteristics:
1) It is based on a single phase, positive sequence model of the transmission system. Note that this modeling assumption was a reasonable simplification that was made due to computational power constraints when state estimation was introduced in the power systems industry in late 1960’s.
2) The measurement set used at each state estimation execution consists of non-simultaneous measurements.
3) A centralized architecture is used in which all the measurements are collected in a central location.

These characteristics result in several biases of present legacy state estimators. In particular the single phase, positive sequence models cannot capture system asymmetries and unbalanced operation of the system which becomes more prevalent the recent years. In addition, the measurement time skewness reduces the robustness of the state estimator. Finally the centralized architecture results in a large scale problem with long execution times usually in the order of 1-3 minutes.

Synchrophasor Technology – Enabling Enhanced State Estimation

Recent advancements in Synchrophasor Technology and the continuously increasing deployment of Phasor Measurement Units (PMU) open up the possibility for the development of enhanced State Estimators that eliminate the biases described before. PMUs provide synchronized measurements with a common, globally valid time reference (UTC time) with magnitude precision in the order of 0.1% and time precision of 1 μsec translating in 0.02 Degrees at 60 Hz. With synchrophasor technology SE can be reformulated both algorithmic and architectural to take advantage of the characteristics of the new technology.

Elimination of State Estimation Biases

Next, the way Synchrophasor Technology can contribute to eliminate the legacy state estimation biases is described:

- PMUs provide three-phase measurements. A three-phase formulated SE can capture system asymmetries and unbalanced operation, thus can eliminate the associated biases of single phase, positive sequence modeling.
- PMUs provide GPS-synchronized and time-tagged phasor measurements, thus the measurement time skewness can be eliminated.
- The computational time of SE can be also improved when using Synchrophasors. If all measurements used in the SE are GPS-synchronized voltage and current phasors and the models are linear then a Linear State Estimation (LSE) can be formulated which has a direct solution. In addition the data quality check and bad data processing can be more effective if synchrophasors are used. Finally GPS-Synchronized measurements make it possible to “distribute” the state estimation process and transition from a centralized architecture to a distributed architecture. Note that the results of a local synchrophasor-based state estimator are “globally” valid.

Different formulations of Synchrophasor-Based State Estimators are described below along with the associated advantages and disadvantages.
Hybrid State Estimation

The state definition in a Hybrid SE is the same as in legacy SE i.e. positive sequence voltage phasors.

The measurement set is expanded by including the voltage and current synchrophasors (magnitudes and phase angles) as additional measurements. This increases the robustness of the SE a) due to increased measurement redundancy and b) since the voltage phase angles are “state measurements”.

As for the solution algorithm, non-linear WLS is used since there is a mix of nonlinear (due to traditional measurements) and linear (due to synchrophasors) equations. A pictorial view of the Jacobian (H) in a Hybrid SE is illustrated in Figure 2.

![Figure 2: Jacobian Matrix in a Hybrid State Estimation Formulation](image)

There are several challenges for a Hybrid SE which are summarized below.

First an angle reference is still required. The power flow slack bus can be used as a reference if a PMU is physically installed in this bus, so all measurement values can be referenced to that. Another option is the selection of the reference angle to be done at the PDC and the phase angle of a specific measurement is selected as reference. This will result in a varying value of the reference which might complicate the operators. In general the selection of the reference angle is challenging since the performance of the SE will depend on the accuracy of the PMU and the measurement that are used as a reference.

Another challenge is the different resolution and synchronization of SCADA and PMU measurements. Synchrophasors are GPS time-tagged using UTC as time reference while SCADA measurements are time-tagged by the EMS clock when they arrive at the control center. In addition Synchrophasors have a higher resolution (usually 10-60 samples/sec) so down sampling is needed.
Given those challenges, the advantages in terms of accuracy and robustness of a Hybrid SE over a legacy SE are questionable and depend on the number of additional synchrophasor measurements and the PMUs location. A very interesting study on the topic conducted as part of an EPRI project with NYPA is documented in [2].

**Linear State Estimation**

The state definition in a Linear SE is the same as in legacy SE i.e. positive sequence voltage phasors.

The measurement set consists only of voltage and current synchrophasors. Thus the measurement equations are linear.

The LSE is also using WLS as the solution algorithm but since the SE formulation is linear then a direct solution can be obtained without the need of iterations, resulting in faster execution of the SE.

The other important thing that has to be emphasized for LSE is that there is no need for a reference angle, since all the angles are already synchronized and referenced to the UTC time. Another advantage of LSE is related to bad data analysis. If a WLS is used then still the “Largest Normalized Residual Hypothesis Test” is used for bad data analysis and the benefits stem from the fact that the solution of each individual state estimation is faster. If a Least Absolute Value solution algorithm is used then bad data analysis is inherent in the linear programming solution which makes the bad data analysis very fast since no iterations are needed. More information on this approach can be found in [3].

Another important topic related to synchrophasor-only SE is observability. For an LSE the observable states are defined by the location of the PMUs. Measurement redundancy is very important since without it the LSE results can be questionable, if, for example, a PMU providing a critical measurement is malfunctioning. In general bad data in critical measurements cannot be detected. If the system is not fully observable, then islands are formed but the advantage here is that these islands are synchronized. The topic of optimal PMU placement to achieve full system observability with the minimal number of PMUs and other objectives such as a specific redundancy level is well established in the literature. Note though that PMU placement has many practical and techno-economic constraints.

![Figure 3: PMU Location and Observability](image-url)
Distributed State Estimation

A State Estimator is distributed if it is performed based on a decentralized architecture and can be implemented either at an area or at a substation level. The advantages of a Distributed State Estimation are the following:

- Reduced dimensionality. Faster computational performance.
- Facilitates use of more accurate models (three phase, dynamic).
- Reduced communications burden and associated time latencies.
- Easier data validation. Easier bad data detection, identification and rejection.

GPS-Synchronized measurements make it possible to “distribute” the state estimation process without the need of additional state estimation for coordination. Note that the results of a local state estimator are “globally” valid if there is at least one valid GPS-synchronized datum.

For an area implementation the state is defined as the set of the internal states and the states of the neighboring buses, as illustrated in Figure 4. Note that PMUs are required at the boundary buses for the neighboring buses to be observable.

\[
x_i = \begin{bmatrix} x_{i, \text{internal}} \\ x_{i, \text{neighboring}} \end{bmatrix}
\]

**Figure 4: Area Level Implementation of a Distributed State Estimator**

The computed state vector from each area is the only information that is sent to the central location. There is no need for measurements to be communicated. The system state vector is synthesized from the individual areas’ state vector without need of additional state estimation at the central location. Accuracy cross-check of the boundary buses state estimates can be performed at the central location. Another advantage is that bad data analysis is easier at the area level. Note that the decentralized architecture can
be also implemented with only SCADA measurements but then an additional central state estimation for coordination is needed.

A Distributed State Estimation can be also implemented at the substation level. In this case the state is defined as the voltage phasor at each bus of the substation and at the boundary bus of the neighboring substations. The main advantages of this approach are:

- Facilitates three-phase and dynamic state estimation formulation since only the model of the substation is needed
- Easier data and model validation due to the small model size. This is also facilitated by the big measurement redundancy that typically holds in a substation. Bad data analysis is also easier and faster if performed separately for each substation.
- Takes advantage of substation automation

More information on substation-level Distributed SE can be found in [4].

![Figure 5: Substation Level Implementation of a Distributed State Estimator [5]](image)

Three-Phase State Estimation

As it was mentioned earlier, legacy SE assumes balanced operation of the system and uses positive sequence network model and measurements. Actual power system operates near balanced conditions and is not perfectly symmetric. A three-phase SE formulation using, three-phase synchrophasor measurements and detailed three-phase asymmetrical network modeling can eliminate legacy state estimation biases and can capture system unbalanced operation and system asymmetries.

The state is defined as the set of individual phase voltage phasors of system's buses.

$$[\mathbf{x}] = \begin{bmatrix} \tilde{V}_{1,A} & \tilde{V}_{1,B} & \tilde{V}_{1,C} & \cdots & \tilde{V}_{n,A} & \tilde{V}_{n,B} & \tilde{V}_{n,C} \end{bmatrix}$$

The measurement set consists of three-phase voltage and current synchrophasors. Thus the measurement equations are linear.
\[
\begin{bmatrix}
\ldots & \tilde{V}_x^m & \tilde{V}_x^m & \tilde{V}_x^m & \ldots & \tilde{I}_{x-y,A}^m & \tilde{I}_{x-y,B}^m & \tilde{I}_{x-y,C}^m & \ldots
\end{bmatrix} = \tilde{H} \cdot [x] + [e]
\]

The main disadvantage of this formulation is that the size of the SE problem increases so distributed implementation is needed for acceptable computational performance.

Three-phase state estimation could be also applied using modal decomposition theory and symmetrical component network modeling. In this case the three-phase measurements are transformed into their symmetrical components \( \tilde{V}_x^m, \tilde{V}_x^m, \tilde{V}_x^m, \tilde{I}_{xy}^m, \tilde{I}_{xy}^m, \tilde{I}_{xy}^m \) and then the SE is solved individually for each symmetrical component. Then the estimates are transformed back from symmetrical components to individual phases. The main disadvantage of this approach is that the system asymmetry is not captured since for the computation of the sequence network values the simplification that all the diagonals elements and all the non-diagonal elements of the impedance matrices are equal.

**Dynamic State Estimation**

A SE is classified as Dynamic if the system model is dynamic i.e. includes differential equations. Dynamic SE has been researched since the 70’s but the initial interest was limited due to limited applicability. The main challenges for the application of Dynamic SE are:

1. Measurements resolution and time alignment
2. Model accuracy
3. Computational performance

Synchrophasor technology facilitates application of Dynamic SE and recently the research community has renewed interest on the topic.

A generic form of the dynamic system model in a Dynamic State Estimation is:

\[
\frac{dx(t)}{dt} = f(x(t), y(t), t) \quad 0 = g(x(t), y(t), t)
\]

where \( x(t) \) are the dynamic states and \( y(t) \) are the algebraic states. Examples of dynamic states are generator torque angle, generator frequency, internal control variables of devices etc. Examples of algebraic states are voltage magnitude, voltage phase angle, internal states of devices etc.

The measurement set may consist of traditional measurements, synchrophasors and additional measurements such as frequency and rate of frequency.

Kalman filter is the most commonly used solution algorithm for Dynamic SE. The basic assumptions of Kalman filter theory are:

- System noise and measurement noise are Gaussian
- System model and measurement model are linear

An optimal solution is obtained under these assumptions. Kalman filter is a two-step algorithm, as shown in Figure 6, with the two steps being:

1. **Prediction Step:** estimates state variables and their uncertainties using the system model
2. **Correction Step:** updates state variables using measurement set. Gives more weight to states with higher certainty.
Other more complex filtering algorithms have been recently researched by PNNL and a very good summary is given in [5]. A high level evaluation of the different algorithms is illustrated in Figure 7.

<table>
<thead>
<tr>
<th></th>
<th>Extended Kalman Filter</th>
<th>Unscented Kalman Filter</th>
<th>Ensemble Kalman Filter</th>
<th>Particle Filter</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Accuracy</strong></td>
<td>The 2\textsuperscript{nd} best with 0% diverged</td>
<td>33% diverged</td>
<td>The best with 0% diverged</td>
<td>20% diverged (PF 2000)</td>
</tr>
<tr>
<td><strong>Efficacy of interpolation</strong></td>
<td>High</td>
<td>High</td>
<td>Low</td>
<td>High</td>
</tr>
<tr>
<td><strong>Number of samples needed</strong></td>
<td>None</td>
<td>Small</td>
<td>Medium</td>
<td>Large</td>
</tr>
<tr>
<td><strong>Sensitivity to missing data</strong></td>
<td>Low</td>
<td>Low</td>
<td>Low</td>
<td>Low</td>
</tr>
<tr>
<td><strong>Sensitivity to outliers</strong></td>
<td>Low</td>
<td>Low</td>
<td>Medium</td>
<td>High</td>
</tr>
<tr>
<td><strong>Computation time (non-parallel)</strong></td>
<td>Shortest</td>
<td>Same order as EKF</td>
<td>Longer than EKF</td>
<td>Same order as EnKF</td>
</tr>
</tbody>
</table>

Figure 7: Evaluation of Different Filtering Algorithms for Dynamic State Estimation [6]

WLS can be also used as a solution algorithm for Dynamic SE. In this case, at each execution step of the algorithm the differential equations are integrated using an integration method such as the trapezoidal rule. Integration transforms all the equations in static and then the methodology as described before in the “State Estimation” section can be used. Note that the same rules with respect to linear vs nonlinear formulation apply also here, depending on the system model and the availability of measurements. More information on this approach can be found in [6].

The main advantage of Dynamic SE is that it is suitable for monitoring generator and load dynamics. In addition it enables estimation of device internal not measurable variables. However the main disadvantage is that it is more sensitive to numerical issues. For a wide area application of Dynamic SE, a centralized architecture implementation is very challenging since it requires very small communication delays and significant computational efficiency. That is why a distributed architecture as described before makes more sense. Potential Dynamic SE applications are in Protection and Control, and real-time dynamic stability assessment.

**Conclusions**
Present legacy State Estimators are based on 1970’s technology and have several biases. Synchrophasor Technology enables advancements in State Estimation both Algorithmic and Architectural. The goal of this White Paper is to describe the advancements in SE that Synchrophasor Technology enables. The formulations of several synchrophasor-based State Estimators have been described, i.e. hybrid, linear, distributed, three-phase and dynamic. A high-level evaluation of those, describing their advantages and challenges, is shown in the summary table in Figure 8.

<table>
<thead>
<tr>
<th>SE</th>
<th>Uses PMU Data</th>
<th>Accuracy</th>
<th>Solution Speed</th>
<th>Technology Readiness Level</th>
</tr>
</thead>
<tbody>
<tr>
<td>Legacy</td>
<td></td>
<td>Single phase, positive sequence models, time skewness</td>
<td>Iterative Solution</td>
<td>Commercial products</td>
</tr>
<tr>
<td>Linear</td>
<td></td>
<td>Phase angle measurements, time-tagged</td>
<td>Direct Solution</td>
<td>Ongoing demos</td>
</tr>
<tr>
<td>Hybrid</td>
<td></td>
<td>Limited, if any, improvements</td>
<td>Iterative Solution</td>
<td>Commercial products</td>
</tr>
<tr>
<td>Three-Phase</td>
<td></td>
<td>Captures system imbalances &amp; asymmetries</td>
<td>Increases problem size</td>
<td>Few demos</td>
</tr>
<tr>
<td>Distributed</td>
<td></td>
<td>Facilitates use of detailed models</td>
<td>Reduces problem size</td>
<td>Few demos</td>
</tr>
<tr>
<td>Dynamic</td>
<td></td>
<td>Captures system dynamics – Numerically sensitive to model accuracy</td>
<td>Computationally challenging – increases problem size</td>
<td>Few demos, still in R&amp;D level</td>
</tr>
</tbody>
</table>

Figure 8: State Estimation Technology Summary Table

References